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International Journal of Heat and Mass Transfer 43 (2000) 3245–3251

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Technical Note

# Mixed convection in film condensation from isothermal vertical surfaces — the entire regime

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Received 13 August 1999; received in revised form 15 October 1999

## 1. Introduction

Condensation heat transfer along isothermal vertical surfaces has many engineering applications in heat exchanger type equipment. The problem of laminar film condensation of water vapor with and without noncondensable gases on an isothermal vertical surface was studied by Minkowycz [1,2] and Fujii [3], just to name a few. However, Minkowycz's study neglected the convective and inertia effects in the liquid layer as well as the shear stress at the interface. A good review of literature and a study on pure forced and pure free convection may be found in Ref. [3]. In mixed convection, the boundary layers become nonsimilar. A limited number of studies [4–11] have been done on mixed convection condensation and no study has covered the entire regime from pure forced convection to pure free convection with the use of a single nonsimilarity parameter. Recently, Shu and Wilks [5,6] studied condensation of saturated vapor along a vertical plate in mixed convection by perturbation series methods and modified the Cebeci box method [12] to handle solutions of the coupled differential equations. Also, Winkler et al. [13] solved the entire regime of mixed convection by dividing it into two regions, one for free convection dominated conditions and another for forced convection dominated conditions.

In the present paper, a single nonsimilar parameter is used to model the entire regime of mixed convection. This parameter, $\chi$, ranges from 0 for pure free convection to 1 for pure forced convection. The governing equations are transformed into dimensionless form by use of the nonsimilar transformation. The local nonsimilarity method, as described in Ref. [14], in conjuncture with a finite difference method is used to solve the system of differential equations. Fluid property data was taken from textbooks [15,16] and from handbook [17].

## 2. Analysis

Consider condensation of a pure vapor along an impermeable isothermal vertical plate in mixed convection, as shown in Ref. [13]. The gravitational acceleration, $g$, is acting downward in the direction of the flow. The flow is assumed to be steady, laminar, and two-dimensional. A condensate film of thickness $\delta$ is then formed adjacent to and along the surface, and a vapor layer of thickness $\delta^*$ exists between the condensate film and the vapor bulk in the liquid–vapor two-phase boundary layer flow. The surface of the plate is maintained at a uniform temperature $T_w$ that is lower than the vapor temperature. The fluid properties are assumed to be constant except for the variations in density which induce the buoyancy force. By employing the laminar boundary layer approximations the

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PII: S0017-9310(99)00341-5

**Nomenclature**

| | |
|---|---|
| $f$, $F$ | dimensionless stream function for liquid and vapor layers, respectively |
| $h(x)$ | local heat transfer coefficient |
| $k$ | thermal conductivity |
| $Gr_x$ | local Grashof number, $gx^3/\nu^2$ |
| $Nu_x$ | local Nusselt number, $hx/k$ |
| $Pr$ | Prandtl number, $\nu/\alpha$ |
| $Re_x$ | local Reynolds number, $u_\infty x/\nu$ |
| $q_w$ | local surface heat flux |
| $T$ | temperature |
| $T_\infty$ | free stream temperature |
| $T_w$ | wall temperature |
| $u$, $v$ | velocity components in $x$- and $y$-direction |
| $u_\infty$ | free stream velocity |
| $x$, $y$ | axial and normal coordinates |
| $\alpha$ | thermal diffusivity |
| $\delta$ | boundary layer thickness |
| $\eta$ | pseudo-similarity variable |
| $\theta$ | dimensionless temperature |
| $\mu$ | dynamic viscosity |
| $\nu$ | kinematic viscosity |
| $\chi$ | nonsimilar parameter, $Re_x^{1/2}/(Re_x^{1/2}+Gr_x^{1/4})$ |
| $\rho$ | fluid density |
| $\psi$, $\Psi$ | stream functions for the liquid and vapor layer, respectively |

*Subscripts*

| | |
|---|---|
| $\infty$ | free stream condition |
| w | wall condition |
| i | interfacial condition |

*Superscripts*

| | |
|---|---|
| $*$ | vapor layer quantity |
| $'$ | partial differentiation with respect to $\eta$, $\eta^*$ for the liquid and vapor layers, respectively |

governing conservation equations for the problem under study are exactly similar to those in Ref. [13].

The governing equations may be transformed from the $(x, y)$ coordinates to the dimensionless coordinates $[\chi(x), \eta(x, y)]$ and $[\chi(x), \eta^*(x, y)]$ for the liquid and vapor layers, respectively, by introducing

$$\eta = (y/x)Re_x^{1/2}\chi^n, \quad \Psi = \nu Re_x^{1/2}\chi^n f(\chi, \eta) = (\nu u_\infty x)^{1/2}\chi^n f(\chi, \eta) \tag{1}$$

$$\eta^* = \left(u_\infty/\nu^*\right)^{1/2}(y-\delta)x^{-1/2}\chi^n, \quad \Psi = \nu^* Re_x^{1/2}\chi^n F\left(\chi, \eta^*\right) = (\nu^* u_\infty x)^{1/2}\chi^n F\left(\chi, \eta^*\right) \tag{2}$$

$$\chi(x) = Re_x^{1/2}/\left(Re_x^{1/2}+Gr_x^{1/4}\right), \quad \theta = \theta^* = (T-T_w)/\left(T_\infty^*-T_w\right) \tag{3}$$

Substituting Eqs. (1)–(3) into the governing equations, one can obtain the following system of equations

$$f''' + \frac{1}{4}(3-\chi)ff'' - \frac{1}{2}(1-\chi)f'^2 + (1-\chi)^4 = \frac{1}{4}\chi(1-\chi)\left[f''\frac{\partial f}{\partial \chi} - f'\frac{\partial f'}{\partial \chi}\right] \tag{4}$$

$$\frac{\theta''}{Pr_w} + \frac{1}{4}(3-\chi)f\theta' = \frac{1}{4}\chi(1-\chi)\left[\theta'\frac{\partial f}{\partial \chi} - f'\frac{\partial \theta}{\partial \chi}\right] \tag{5}$$

$$F''' + \frac{1}{4}(3-\chi)FF'' - \frac{1}{2}(1-\chi)F'^2 + \left[\frac{\rho\rho^* - \rho\rho_\infty^*}{\rho\rho^* - \rho^*\rho_\infty^*}\right](1-\chi)^4 = \frac{1}{4}\chi(1-\chi)\left[F''\frac{\partial F}{\partial \chi} - F'\frac{\partial F'}{\partial \chi}\right] \tag{6}$$

$$\frac{\theta^{*\prime\prime}}{Pr_\infty^*} + \frac{1}{4}(3-\chi)F\theta^{*\prime} = \frac{1}{4}\chi(1-\chi)\left[\theta^{*\prime}\frac{\partial F}{\partial \chi} - F'\frac{\partial \theta^*}{\partial \chi}\right] \tag{7}$$

It should be noted that it was found that $n = -1$ and the following useful expression was derived from the definition of $\chi$, $Gr_x = gx^3/\nu^2$ and $Re_x = u_\infty x/\nu$: $\chi \mathrm{d}\chi/\mathrm{d}x = -\chi(1-\chi)/4$.

The boundary conditions become the following

$$f'(\chi, 0) = 0, \quad (3-\chi)f(\chi, 0) - \chi(1-\chi)\frac{\partial f}{\partial \chi}(\chi, 0) = 0, \quad \theta(\chi, 0) = 0 \quad f'(\chi, \eta_i) = F'(\chi, 0) \tag{8}$$

$$R_{prop}\left[(3-\chi)f(\chi, \eta_i) - \chi(1-\chi)\frac{\partial f}{\partial \chi}(\chi, \eta_i)\right] = (3-\chi)F(\chi, 0) - \chi(1-\chi)\frac{\partial F}{\partial \chi}(\chi, 0) \tag{9}$$

$$R_{\mathrm{prop}} f''(\chi, \eta_{\mathrm{i}}) = F''(\chi, 0),$$

$$\theta(\chi, \eta_{\mathrm{i}}) = \theta^*(\chi, 0) = \theta_{\mathrm{i}} \tag{10}$$

$$\theta'(\chi, \eta_{\mathrm{i}}) = \left(k^*/k\right)\left(\nu/\nu^*\right)^{1/2} \theta^{*\prime}(\chi, 0) + \left[\frac{1}{4}(3-\chi) f(\chi, \eta_{\mathrm{i}}) - \frac{1}{4}\chi(1-\chi)\frac{\partial f}{\partial \chi}(\chi, \eta_{\mathrm{i}})\right] \times \frac{Pr_{\mathrm{w}} h_{\mathrm{fg}}}{C_{\mathrm{p}}\left(T_\infty^* - T_{\mathrm{w}}\right)} \tag{11}$$

$$F'(\chi, \infty) = \chi^2, \quad \theta^*(\chi, \infty) = 1 \tag{12}$$

The primes denote partial differentiation with respect to $\eta$ and $\eta^*$ for the liquid and vapor layers, respectively. The viscous parameter $R_{\mathrm{prop}}$ can be expressed as the following: $R_{\mathrm{prop}} = [\rho\mu/(\rho^*\mu^*)]^{1/2}$.

Some of the physical quantities of interest include the velocity components $u$ and $v$ in the $x$- and $y$-directions, the wall shear stress $\tau_{\mathrm{w}}$, defined as $\tau_{\mathrm{w}} = \mu(\partial u/\partial y)_{y=0}$, the local Nusselt number $Nu_x = hx/k$, where $h = q_{\mathrm{w}}/(T_{\mathrm{w}} - T_{\mathrm{i}})$, and the condensate mass flux, $\dot{m}_x$. They are given by

$$u = u_\infty \chi^{-2} f'(\chi, \eta), \quad u^* = u_\infty \chi^{-2} F'\left(\chi, \eta^*\right) \tag{13}$$

$$v = -\frac{\nu}{x} Re_x^{1/2} \frac{1}{\chi}\left[\frac{1}{4}(3-\chi) f(\chi, \eta) - \frac{1}{4}\eta(1+\chi) f'(\chi, \eta) - \frac{1}{4}\chi(1-\chi)\frac{\partial f}{\partial \chi}(\chi, \eta)\right] \tag{14}$$

$$v^* = -\frac{\nu^*}{x} Re_x^{*1/2} \frac{1}{\chi}\left[\frac{1}{4}(3-\chi) F\left(\chi, \eta^*\right) - \frac{1}{4}\left(\frac{Re_x^*}{Re_x}\right)^{1/2} \eta(1+\chi) F'\left(\chi, \eta^*\right) - \frac{1}{4}\chi(1-\chi)\frac{\partial F}{\partial \chi}\left(\chi, \eta^*\right)\right] \tag{15}$$

$$\tau_{\mathrm{w}} = (\mu/x) u_\infty \chi^{-3} Re_x^{1/2} f''(\chi, 0) \tag{16}$$

$$Nu_x\left(Re_x^{1/2} + Gr_x^{1/4}\right)^{-1} = \theta'(\chi, 0)/\theta_{\mathrm{i}} \tag{17}$$

$$\dot{m}_x\left(Re_x^{1/2} + Gr_x^{1/4}\right)^{-1}(x/\mu) = \frac{1}{4}(3-\chi) f(\chi, \eta_{\mathrm{i}}) - \frac{1}{4}\chi(1-\chi)\frac{\partial f}{\partial \chi}(\chi, \eta_{\mathrm{i}}) \tag{18}$$

Now we will examine the density ratio in Eq. (6). Let this ratio be denoted as $W$ which has the following expression: $W = (\rho\rho^* - \rho\rho_\infty^*)/(\rho\rho^* - \rho^*\rho_\infty^*)$. Note that this ratio will be equal to zero for saturated vapors since $\rho^* = \rho_\infty^*$ for saturated vapors. For superheated vapor, $W \neq 0$ and it may prove difficult to express $W$ accurately for a variable property case with superheated vapors.

## 3. Methods of solution

In order to apply the local nonsimilarity method, one takes $\partial/\partial\chi$ of the governing equations and their boundary conditions to obtain a system of equations for $\partial f/\partial\chi$, $\partial F/\partial\chi$, $\partial\theta/\partial\chi$, $\partial\theta^*/\partial\chi$, and their boundary conditions. Terms involving $\chi\partial^2 f/\partial\chi^2$, $\chi\partial^2 F/\partial\chi^2$, $\chi\partial^2\theta/\partial\chi^2$, $\chi\partial^2\theta^*/\partial\chi^2$ are then neglected in the latter set of equations. This results in a system of equations for $f$, $F$, $\theta$, $\theta^*$, $\partial f/\partial\chi$, $\partial F/\partial\chi$, $\partial\theta/\partial\chi$, $\partial\theta^*/\partial\chi$, along with the corresponding boundary conditions (see, for example, [14]). For a given $\eta_{\mathrm{i}}$, a guess at the value of $f'(\chi, \eta_{\mathrm{i}})$ is made and a finite difference method is used to solve the $f$ and $\partial f/\partial\chi$ equations.

Next, the $F$ and $\partial F/\partial\chi$ equations may be solved using the same finite difference method. The convergence criteria for the method used is as follows. First, the iterations are stopped when successive iterations of the shear at the wall and the shear at the interface are within $10^{-6}$. Next, the solution is assumed correct when the condition $|R_{\mathrm{prop}} f''(\chi, \eta_{\mathrm{i}}) - F''(\chi, 0)| \leq 10^{-3}$ is met. If this condition is not met, a new guess of $f'(\chi, \eta_{\mathrm{i}})$ is made and the process repeated until the shear stress condition at the interface is met. This generally occurs within 10 iterations. Next, a solution to the $\theta$ and $\partial\theta/\partial\chi$ may be carried out. A solution to the $\theta^*$ and $\partial\theta^*/\partial\chi$ is not needed since only saturated vapors are being considered. Iterations on the $\theta$ and $\partial\theta/\partial\chi$ systems are stopped when the difference in the heat flux at the wall between the two successive iterations are within $10^{-6}$. Step sizes of $\Delta\eta = 0.01$, $\Delta\eta^* = 0.02$ and $\Delta\chi = 0.02$ were used.

## 4. Results and discussion

Representative numerical results for saturated steam and R-134a at one atmospheric pressure are presented in this section. By fixing the pressure, the temperature of the condensate at the interface will be equal to its saturation temperature at that pressure. A wall temperature of 80°C was chosen for all steam calculations and −40°C was chosen for all refrigerant calculations. Since a constant property model is not physically realistic, a reference temperature must be chosen to evaluate physical properties. The reference temperature chosen was that used by Minkowycz [1] and is given

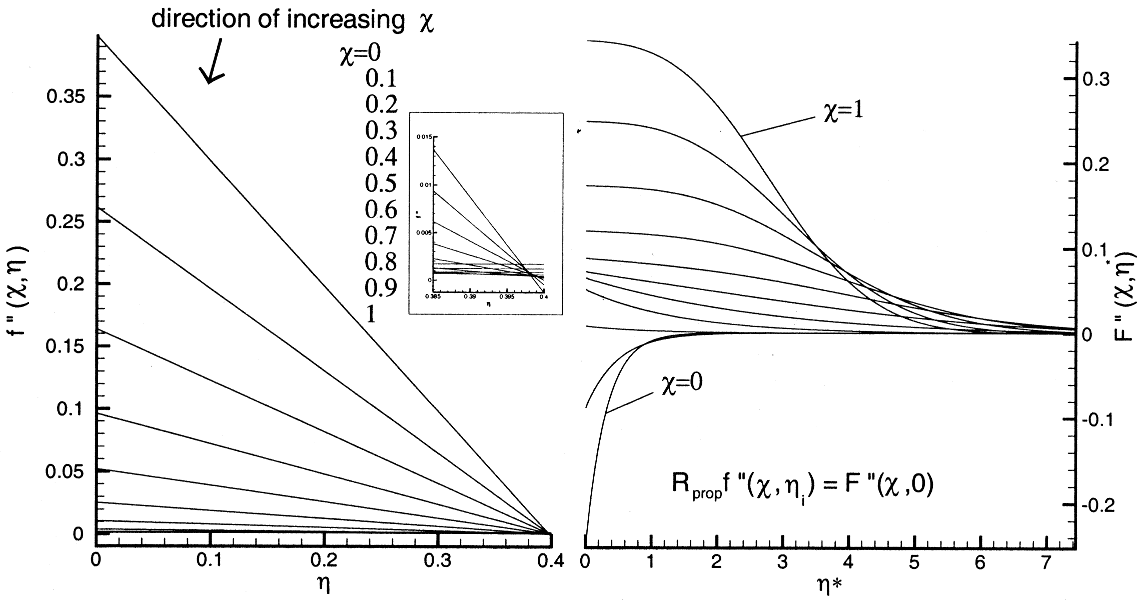


Fig. 1. $f''$ and $F''$ profiles for saturated steam, $R_{\mathrm{prop}} = 200$, for the entire regime of mixed convection.

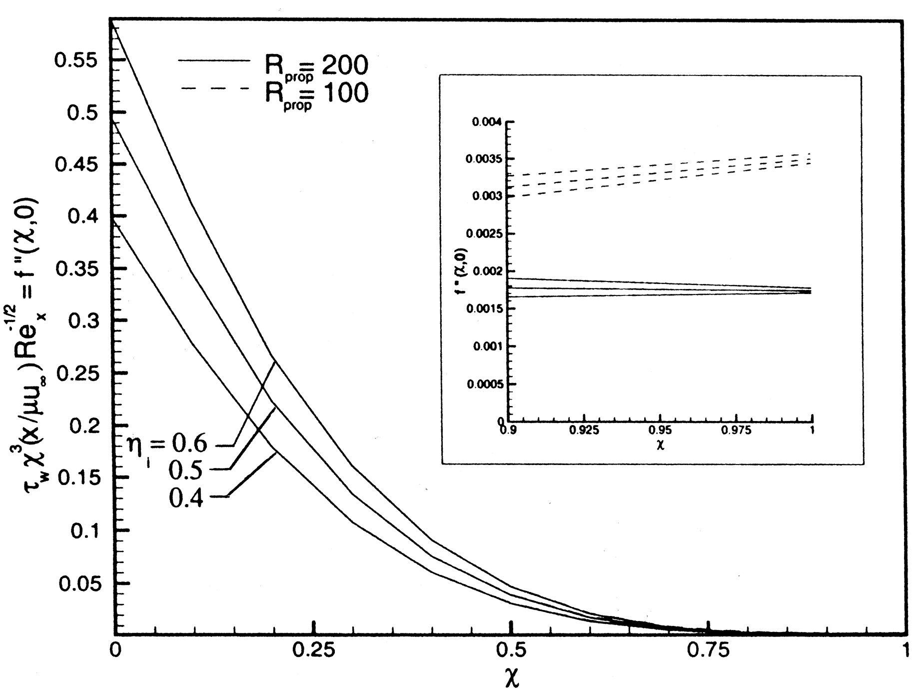


Fig. 2. $f''(\chi, 0)$ values for saturated vapor for the entire regime of mixed convection.

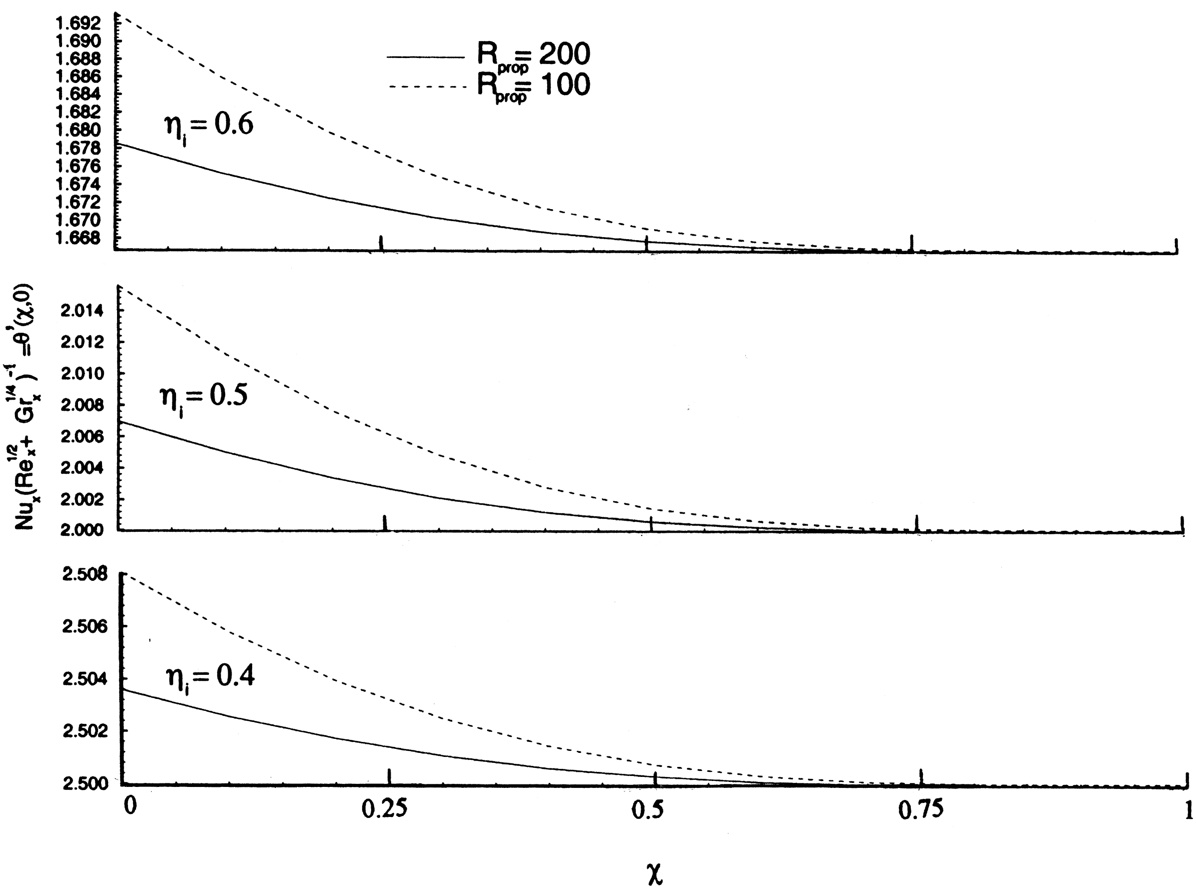


Fig. 3. $Nu_x(Re_x^{1/2} + Gr_x^{1/4})^{-1}$ for saturated vapor for the entire regime of mixed convection.

by

$$T_{\mathrm{reference}} = T_{\mathrm{w}} + 0.31(T_{\mathrm{i}} - T_{\mathrm{w}}) \tag{19}$$

This rule for the liquid layer brings constant property heat transfer results to within ±0.3% of those obtained from variable property solutions for free convection. It is not known how the constant-property results will compare with variable-property results in mixed convection. A value of $R_{\mathrm{prop}} = 200$ was used for saturated steam, with $Pr_{\mathrm{w}} = 2.25$. For R-134a, $R_{\mathrm{prop}} = 100$ and $Pr_{\mathrm{w}} = 5$ were used. The Prandtl number for the vapor layer is not needed since only saturated vapors are being considered.

The velocity and temperature profiles, $f'(\chi, \eta)$ and $\theta(\chi, \eta)$, for saturated steam are shown in [18], and omitted here to save space. A representative film thickness of $\eta_{\mathrm{i}} = 0.4$ was chosen for plotting the results. Fig. 1 shows the shear stress distribution throughout the liquid and vapor layers. As one can see, there is significant dimensionless shear stress at the interface which proves that the common assumption of a shear free interface is inappropriate. The behavior of the velocity, temperature, and shear profiles for the refrigerant are similar, and they are therefore not shown to conserve space.

To find the local Nusselt number and the local wall shear stress, one needs to know the values of $\theta'(\chi, 0)$ and $f''(\chi, 0)$. These quantities at selected values of $\chi$ are listed in [18] for steam and R-134a for $\eta_{\mathrm{i}} = 0.4$, 0.5 and 0.6. As illustrated in Fig. 2, the dimensionless wall shear stress can be seen to increase by two orders of magnitude from pure forced convection ($\chi = 1$) to pure free convection ($\chi = 0$). However, in Fig. 3, one can see that the Nusselt number parameter only increases slightly with a decrease in $\chi$ from 1 to 0. At first glance of the results, it may seem that the actual Nusselt number is greater for pure free convection and pure forced convection than it is for mixed convection. However, this is not the case. Consider, for example, $R_{\mathrm{prop}} = 200$, $Pr_{\mathrm{w}} = 2.25$, $\eta_{\mathrm{i}} = 0.5$ and $\chi = 0.9$. If the Reynolds number is taken as $Re_x = 1000$, the corre-

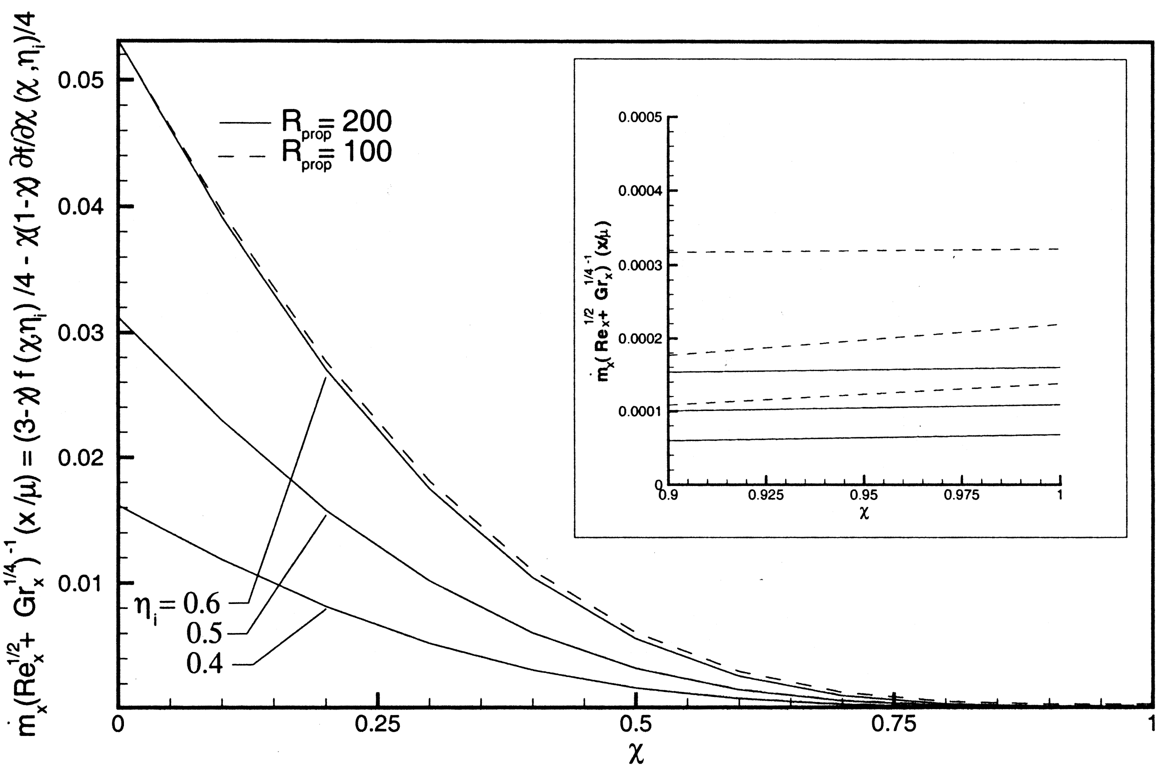


Fig. 4. Dimensionless mass flux for saturated vapor for the entire regime of mixed convection.

sponding Grashoff number can be found from the $\chi$ expression to be $Gr_x = 152.4$. Using the $\theta'(\chi, 0)$ values listed in [18], the local Nusselt number for mixed convection ($Re_x = 1000$, $Gr_x = 152.4$) is 70.3, but for pure forced convection ($Re_x = 1000$) and pure free convection ($Gr_x = 152.4$), the corresponding Nusselt numbers are found to be 63.2 and 7.1, respectively. From these results, it is obvious that the predicted value of the local Nusselt number for mixed convection is higher than that for pure forced convection and pure free convection. Fig. 2 may also be misleading at first glance. For the mixed convection conditions mentioned above, the dimensionless shear stress parameter, $\tau_w x^2/(\mu\nu)$, has the value of 56.5, while for pure forced convection and pure free convection this quantity is 55 and 21.5, respectively, illustrating that the wall shear stress is higher for mixed convection than for pure forced and pure free convection.

The condensate thickness, as expressed in terms of $\eta_i$, affects both the wall shear stress and wall heat flux, as can be seen in [18]. For both fluids, the wall heat flux is seen to decrease with an increase in $\eta_i$, while the wall shear stress is shown to increase with an increase in $\eta_i$. A physical explanation for this has already been given in [13].

The last quantity of interest is the condensate mass flux, which is shown in Fig. 4 for both $R_{prop} = 200$ and 100. It can clearly be seen from the figure that an increase in $\chi$ causes a decrease in the dimensionless condensate mass flux. However, near $\chi = 1$, the dimensionless condensate mass flux is slightly greater than that of $\chi$ values near 1, say, $\chi = 0.9$. Just as in the Nusselt number argument above, this does not imply that the actual condensate mass flux is lower for mixed convection.

Finally, to give an idea of the accuracy of the method used, results for $\chi = 0$ and 1 were compared to those obtained by the Runge–Kutta method since for these two cases the equations reduce to ordinary differential equations. With $\eta_i = 0.4$ and $R_{prop} = 200$, the Runge–Kutta method gives $f''(0, 0) = 0.3983$ and $\theta'(0, 0) = 2.5034$ for $\chi = 0$, and $f''(1, 0) = 0.00174$ and $\theta'(1, 0) = 2.5000$ for $\chi = 1$. These values differ less than 1.8% from those listed in [18]. These values also compare well with the previous study of Winkler et al. [13]. The present results are compared with those of Shu and Wilks [5]. They employed the nonsimilar parameter $\xi = gx/u_\infty^2$. Their value of $\xi = 0.2$, for instance, corresponds to a value of $\chi = 0.599$. For $Pr_w = 10$, $R_{prop} = 10$, and $\eta_i = 1.04$, Shu and Wilks [5] give $Nu_x Re_x^{-1/2} = 1.6902$, whereas the present analysis for these parametric values gives $Nu_x Re_x^{-1/2} = 1.7046$, which is 0.9% off from their result. This also compares well with $Nu_x Re_x^{-1/2} = 1.7089$ given by Winkler et al. [13].

## 5. Concluding remarks

Condensation from a vertical flat plate in mixed convection was studied analytically. The analysis introduced a new nonsimilarity variable $\chi$, which ranges from zero for pure free convection to one for pure forced convection. The parameter $\chi$ was found to have the expression $\chi = Re_x^{1/2}/(Re_x^{1/2} + Gr_x^{1/4})$. Results for saturated steam and R-134a were presented for representative film thicknesses. It has been found that mixed convection significantly increases the dimensionless wall shear stress and the dimensionless condensate mass flux, but to a lesser degree, it also increases the dimensionless wall heat flux. A comparison of the present results with others has led to the conclusion that the present approach is comprehensive, accurate and simple, and is therefore superior to earlier studies.

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